Chiral News

APCTP workshop "HiSGRA and its Applications", Pohang

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FREEDOM TO RESEARCH

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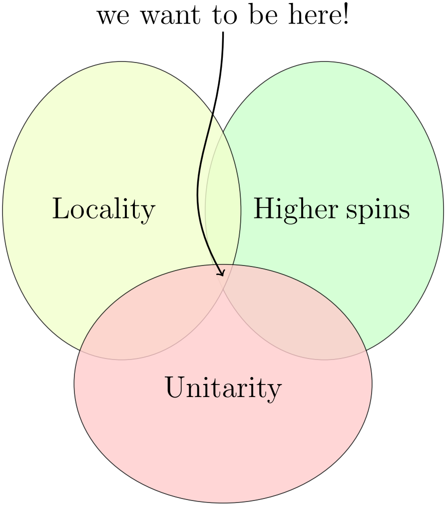
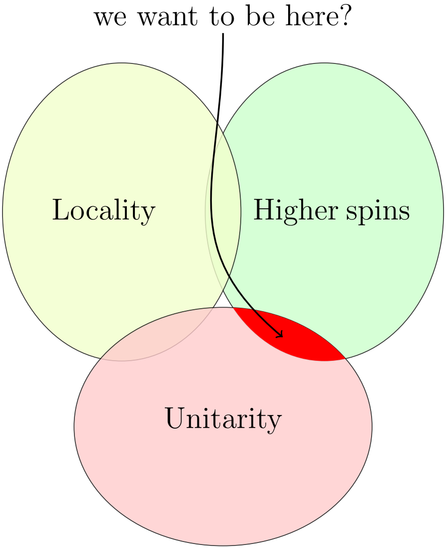
Locality, unitarity, higher spins in an ideal world © **Mkrtchyan**



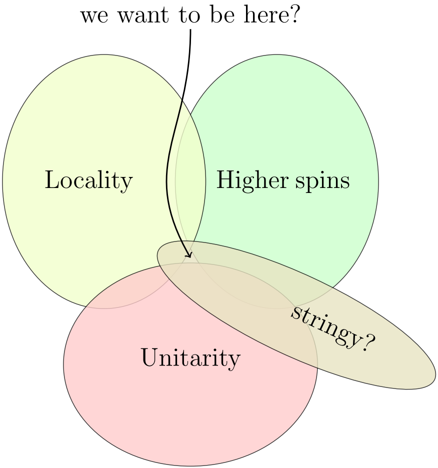
Dream: it would be great to have many local, unitary higher spin gravities with nontrivial (holographic) S-matrix, describe reality, tensionless strings, ...



There does not seem to be such a theory as of today, neither there is an idea on how to construct such.

There are no clear field theory rules on how to drop locality, but see (Jevicki et al; Aharony et al; Neiman; Ponomarev; ...)

Locality, unitarity, higher spins in the real world

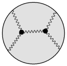


Non-locality → extended objects? (Maldacena, Simmons-Duffin, Zhiboedov; Neiman; ...)

For now: let's study HiSGRA that are field theories

Unitarity → Unitary results

Why massless higher spins should not exist?

	Flat	**(A)dS**
Global	decoupling of longitudinal modes $\delta\Phi_{a_1\dots a_s} = \partial_{(a_1}\xi_{a_2\dots a_s)}$, or tensorial charges $Q_{a_1\dots a_{s-1}}$ impose ∞-many constraints: $\boldsymbol{S = 1}^{**}$ (Weinberg; Coleman, Mandula)	same thing, 50 years later! $\Phi_{\mu_1\dots\mu_s}$ are dual to $J_{a_1\dots a_s}$, $\boldsymbol{S}$ **= free CFT** (Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S., Taronna; Alba, Diab); b.c. give an access to (Chern-Simons) vector models
Local	Noether procedure, i.e. $\mathcal{L} = (\partial\Phi)^2 + \mathcal{O}(\Phi^3)$, $\delta\Phi = \partial\xi + \mathcal{O}(\Phi,\xi)$ is obstructed at Φ^4 (Bekaert, Boulanger, Leclercq; Roiban, Tseytlin; Ponomarev, E.S., ...)	AdS/CFT gives one-line proof $\Phi^4 \sim$  (Erdmenger, Bekaert, {Ponomarev}, (Sleight; Taronna)), which again invalidates Noether procedure at Φ^4

Constraints on (holographic) S-matrix

Asymptotic higher spin symmetries (HSS)

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \nabla_{(\mu_1}\xi_{\mu_2\dots\mu_s)}$$

seem to completely fix (holographic) S-matrix to be

$$S_{\text{HiSGRA}} = \begin{cases} 1^{***}, & \text{flat space,} \\ \text{free CFT,} & \text{AdS, unbroken HSS,} \\ \textbf{Chern-Simons Matter}, & \textbf{AdS}_4\textbf{, slightly-broken HSS} \end{cases}$$

(Sundborg; Klebanov, Polyakov; Sezgin, Sundell; Leigh, Petkou; Maldacena, Zhiboedov; Giombi, Yin, ...)

Trivial/known S-matrix can still be helpful for QG toy-models

The most interesting applications are to three-dimensional dualities (power of HSS is underexplored)

Both Minkowski and AdS cases reveal certain non-localities to be tamed. HSS mixes ∞ spins and derivatives, invalidating the local QFT approach

Basic HS-folklore/mythology, © **Alkalaev**

- Myth/Axiom 1: gauge and gravitational interactions of massless higher spin fields **do not exist in flat space**
- Myth/Axiom 2: anti-de Sitter is very much different from flat space as far as the HS-problem goes, **flat limit is singular**
- Myth/Axiom 3: **generic HiSGRA obey field theory rules**, one just needs to do Noether procedure in AdS and pray hard

Instead: there are no objective facts that would distinguish between flat, (A)dS as far as HS goes: same no-go's, even easier to prove in AdS

All field-theory-type HiSGRA are strange ($3d$ and conformal). One can construct a unique theory with propagating massless fields that exists in flat and $(A)dS_4$, the $(A)dS$-version being a smooth deformation of the flat one. Relation to physics is via vector models and $3d$ dualities

Plan

- Brief summary of Light-front results
- New (long forgotten) covariant description of HS fields
- Contractions of Chiral Theory and gauge/gravitational interactions
- Chiral Theory as FDA in flat/(A)dS
- Applications of Chiral Theory to $3d$ bosonization duality
- Chiral Theory and tensionles strings
- Future prospects
- HS-symmetry and $3d$ bosonization duality (?)

Living on Light-Front

Light-front approach

Good news: Light-front approach deals exclusively with physical degrees of freedom! Bad news: one has to work less hard 

Main principle = more or less the definition of QFT: one has to construct generators P^{μ}, $J^{\mu\nu}$ of Poincare algebra, the only relations to worry being

$$[J^{a-}, P^{-}] = 0 \qquad\qquad \boldsymbol{[J^{a-}, J^{b-}] = 0}$$

Ugly, but pays back: strings, $\mathcal{N} = 4$ SYM, higher spins ....

$4d$ is a special case: $\Phi_{\mu_1 \dots \mu_s}$ boils down to two helicity eigen states, $\Phi_{\pm s}$

$$H \equiv P^{-} = \int \Phi_{-s} E_p \Phi_s + \mathcal{O}(\Phi^3) \qquad\qquad E_p = \frac{\vec{p}_{\perp}^{\,2}}{2p^{+}}$$

Once we know H, we can get an (off-shell) action or evaluate S-matrix

Light-cone approach: interactions

Poincare algebra fixes assuming **locality**

$$H \equiv P^- = \int \Phi_{-s} E_p \Phi_s + V_3(\Phi,\Phi,\Phi) + \dots$$

Flat space story: For any triplet of helicities λ_i, $\lambda_1+\lambda_2+\lambda_3>0$ there is a unique interaction vertex (Bengtsson^2, Linden, 1987):

$$V_3 \sim C_{\lambda_1,\lambda_2,\lambda_3}[12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[31]^{\lambda_3+\lambda_1-\lambda_2}$$

and there is another one for $\lambda_1+\lambda_2+\lambda_3<0$. Plus we have $\Phi_0\Phi_0\Phi_0$.

$(\pm1,\pm1,\mp1)$ gives Yang-Mills vertex; $(\pm2,\pm2,\mp2)$ is Einstein-Hilbert

Surprise 0: $(\pm s,\pm 2,\mp s)$ gives the desired 2-derivative gravitational interaction for any s! One can see the same via spinor-helicity approach, i.e. this is not a 'weird' light-cone feature. **There exists a covariant form!**

Light-cone approach: interactions

Poincare algebra fixes

$$H \equiv P^{-} = \int \Phi_{-s} E_p \Phi_s + V_3(\Phi, \Phi, \Phi) + \dots$$

Flat space story: Coupling constants $C_{\lambda_1,\lambda_2,\lambda_3}$ are not yet fixed

$$V_3 \sim C_{\lambda_1,\lambda_2,\lambda_3} l_p^{\lambda_1+\lambda_2+\lambda_3-1} [12]^{\lambda_1+\lambda_2-\lambda_3} [23]^{\lambda_2+\lambda_3-\lambda_1} [31]^{\lambda_3+\lambda_1-\lambda_2}$$

Metsaev managed to climb up to the quartic level in 1990-1991

Surprise 1: There is a closed system $\sum_\omega C_{\lambda_1,\lambda_2,-\omega} C_{\omega,\lambda_3,\lambda_4} \dots = 0$, which is insensitive to quartic H_4. It has a unique solution $C = 1/\Gamma[\lambda_1+\lambda_2+\lambda_3]$ once at least one genuine HS-interaction is present

Surprise 2: All the couplings, e.g. gravitational, are necessarily present

Summary so far

Vertices: There is a vertex for every triplet λ_i that has $|\lambda_1+\lambda_2+\lambda_3|>0$ derivatives and the scalar self-coupling

Surprise 1: There is 1-to-1 between flat and AdS vertices (Metsaev)

Surprise 2: There are 'low' derivative vertices, e.g. gravitational, that (a) are not writable in terms of Fronsdal fields; (b) are essential for consistency.

Surprise 3: One class of local HiSGRA in flat and $(A)dS$: Chiral with $C=1/\Gamma[\lambda_1+\lambda_2+\lambda_3]$. Another type of flat limit: $R \gg l_p$, which is smooth for all vertices together.

Fradkin-Vasiliev vs. Flat: FV-vertex must be a chimera: $(2s-2)$-derivative combined with 2-derivative with Metsaev coefficient. What is non-smooth is the flat limit of Fronsdal's description

Surprise 4: Noether does not want to work both in flat and AdS due to non-localities. New ideas are welcome! ($3d$, Conformal, Chiral, ..., FDA)

Chiral Higher Spin Gravity on Light-Front

Chiral HSGRA in Minkowski

Self-dual Yang-Mills in Lorentzian signature is a useful analogy

- the theory is non-unitary due to the interactions ($\boldsymbol{A}_\mu \to \boldsymbol{\Phi}^\pm$)

$$\boldsymbol{\mathcal{L}}_{\mathsf{YM}} = \mathbf{tr}\, \boldsymbol{F}_{\mu\nu} \boldsymbol{F}^{\mu\nu}$$

$$\wr\wr$$

$$\boldsymbol{\mathcal{L}}_{\mathsf{SDYM}} = \boldsymbol{\Phi}^- \Box \boldsymbol{\Phi}^+ + \boldsymbol{V}^{++-} + \boldsymbol{V}^{--+} + \boldsymbol{V}^{++--}$$

- tree-level amplitudes vanish, $A_{\mathsf{tree}} = 0$
- one-loop amplitudes do not vanish, are rational and coincide with $(+ + ...+)$ of pure QCD

Loophole: a non-unitary theory that gives only unitary results!

Chiral HiSGRA (Metsaev; Ponomarev, E.S.) is a 'higher spin extension' of SDYM/SDGR. It has fields of all spins $s = 0, 1, 2, 3, ...$:

$$\boldsymbol{\mathcal{L}} = \sum_{\lambda} \Phi^{-\lambda} \Box \Phi^{+\lambda} + \sum_{\lambda_i} \frac{\kappa \, l_{\mathrm{Pl}}^{\lambda_1+\lambda_2+\lambda_3-1}}{\Gamma(\lambda_1+\lambda_2+\lambda_3)} V^{\lambda_1,\lambda_2,\lambda_3}$$

light-cone gauge is very close to the spinor-helicity language

$$V^{\lambda_1,\lambda_2,\lambda_3} \sim [12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[13]^{\lambda_1+\lambda_3-\lambda_2}$$

Locality + Lorentz invariance + genuine higher spin interaction result in a unique completion. **Chiral $\in$ any $4d$ HiSGRA**

This is the smallest higher spin theory and it is unique.
Graviton and scalar field belong to the same multiplet

No UV Divergences! One-loop finiteness

Tree amplitudes vanish. The interactions are naively non-renormalizable, the higher the spin the more derivatives:

$$V^{\lambda_1,\lambda_2,\lambda_3} \sim \partial^{|\lambda_1+\lambda_2+\lambda_3|}\Phi^3$$

but there are **no UV divergences!** (E.S., Tsulaia, Tran). Some loop momenta eventually factor out, just as in $\mathcal{N}=4$ SYM, but ∞-many times.

At one loop we find three factors: (1) SDYM or all-plus 1-loop QCD; (2) higher spin dressing to account for λ_i; (3) total number of d.o.f.:

$$A^{\text{1-loop}}_{\text{Chiral}} = A^{++\ldots+}_{\text{QCD,1-loop}} \times D^{\text{HSG}}_{\lambda_1,\ldots,\lambda_n} \times \sum_\lambda 1$$

$\#$ d.o.f.$= \sum_\lambda 1 = 1 + 2\sum_{\lambda>0} 1 = 1 + 2\zeta(0) = 0$ to comply with no-go's, (Beccaria, Tseytlin) and agrees with many results in AdS, where $\neq 0$

Chiral HSGRA in Minkowski

- **stringy 1**: the spectrum is infinite $s = 0, (1), 2, (3), 4, \dots$
- **stringy 2**: admit Chan-Paton factors, $U(N)$, $O(N)$ and $USp(N)$
- **stringy 3**: we have to deal with spin sums $\sum_s$ (worldsheet takes care of this in string theory) and ζ-function helps
- **stringy 4**: the action contains parts of YM and Gravity
- **stringy 5**: higher spin fields soften amplitudes
- consistent with Weinberg etc. $S = 1^{***}$ (in Minkowski)
- gives all-plus QCD or SDYM amplitudes from a gravity

Apart from Minkowski space the theory exists also in (anti)-de Sitter space, where holographic S-matrix turns out to be nontrivial ... and related to Chern-Simons matter theories

Chiral Higher Spin Gravity: covariantization

What can be done?

- All HSGRA in $4d$ need all possible vertices save for scalar self-coupling, but some couplings are not writable with Fronsdal's fields
- There are some contractions of Chiral HSGRA, which have 1- and 2-derivative interactions — gauge and gravitational (Ponomarev)
- It would be great to find Lorentz covariant approach/form for all the vertices, especially, the gravitational ones in flat space
- It cannot be Fronsdal's, but there are other smart people to choose from. Let's choose Penrose and pals

Vector-spinor dictionary

Each $\mu = 0, ..., 3$ equals AA' where $A, B, ... = 1, 2$ and $A', B', ... = 1, 2$

$$\sigma_\mu^{AA'} v^\mu = v^{AA'} \qquad\qquad v = \begin{pmatrix} t+x & y+iz \\ y-iz & t-x \end{pmatrix}$$

In general we have $V^{A(n),A'(m)}$ and all indices are symmetric. The only anti-symmetric object is invariant $\epsilon_{AB} = -\epsilon_{BA}$, idem. for $\epsilon_{A'B'}$. Abstract Penrose notation:

$$\begin{aligned} \text{Maxwell}: &\qquad F_{\mu\nu} = F_{AB}\epsilon_{A'B'} + \epsilon_{AB}F_{A'B'} \\ \text{Weyl}: &\qquad C_{\mu\nu,\lambda\rho} = C_{ABCD}\epsilon_{A'B'}\epsilon_{C'D'} + \epsilon_{AB}\epsilon_{CD}C_{A'B'C'D'} \\ \text{Traceless}: &\qquad \Phi_{\mu(s)} = \Phi_{A(s),A'(s)} \end{aligned}$$

Any of $V^{A(n),A'(m)}$ with $n+m=2s$ can describe a spin-s field. For $n=m=s$ we have a symmetric/Hermitian description. For $m=2s$, $n=2s$ we have (conjugate) Weyl tensors $\Psi^{A(2s)}$, $\Psi^{A'(2s)}$.

Twistor-inspired approach

Twistors treat positive and negative helicities differently:

$$\nabla_B{}^{A'}\,\Psi^{BA(2s-1)} = 0 \qquad \text{(Penrose, 1965)}$$

$$\nabla^A{}_{B'}\,\Phi^{A(2s-1),B'} = 0 \qquad \delta\Phi^{A(2s-1),B'} = \nabla^{AB'}\xi^{A(2s-2)}$$

(Hitchin, 1980) entertains a possibility to introduce a connection

$$\omega^{A(2s-2)} \ni e_{BB'}\Phi^{A(2s-2)B,B'} \qquad \delta\omega^{A(2s-2)} = \nabla\xi^{A(2s-2)}$$

where $e_{AA'}$ is the vierbein and with $H^{AB} \equiv e^A{}_{C'} \wedge e^{BC'}$ we can write

$$\boldsymbol{S = \int \Psi^{A(2s)} \wedge H_{AA} \wedge \nabla\omega_{A(2s-2)}}$$

which is also invariant under $\delta\omega^{A(2s-2)} = e^A{}_{C'}\,\eta^{A(2s-3),C'}$ to get rid of the extra component. The simplest action for HS.

N.B: for $s = 1$ we have Ψ^{AB} and $A^{CC'}$, for $s = 2$ Ψ^{ABCD} and ω^{AB}

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$$\boldsymbol{S = \int \Psi^{A(2s)} \wedge H_{AA} \wedge \nabla\omega_{A(2s-2)}}$$

Feature: allow us to put higher spins on any self-dual background.

Surprise: presymplectic-AKSZ (Grigoriev et al) naturally contains the same action (E.S., Sharapov) from Hochschild cohomology of HS algebra

Self-dual Theories

- actions are not real in Minkowski space
- actions are simpler than the complete theories
- integrability, instantons (Atiyah, Hitchin, Drinfeld, Manin; ...)
- **SD theories are consistent truncations**, so anything we can compute will be a legitimate observable in the full theory; **any solution of SD is a solution of the full**; ...
- different expansion schemes: instantons instead of flat, MHV, etc.

In general: amplitudes (MHV, BCFW, double-copy, ...), strings, QFT, Twistors, ... encourage to go outside Minkowski

In higher spins: little explored (Adamo, Hähnel, McLoughlin; E.S., Ponomarev; Ponomarev; Tung), can be the only reasonably local theories

Self-dual Yang-Mills

With $F^2_{\mu\nu} = F^2_{AB} + F^2_{A'B'}$ and with $F \wedge F = F^2_{AB} - F^2_{A'B'}$ being topological we can massage YM action

$$S_{YM} = \frac{1}{g^2}\int F^2_{\mu\nu} \sim \frac{1}{g^2}\int F^2_{AB} \sim \int \boldsymbol{\Psi^{AB} F_{AB} - \frac{g'}{2}\Psi^2_{AB}}\,,$$

which is not manifestly real! The first part is the SDYM action

$$S_{\mathsf{SDYM}}[\Psi,\omega] = \int \Psi^{CD} F_{CD}(\omega) = \int \Psi^{CD} H_{CD} \wedge d\omega + \ldots$$

where we see the familiar action

Higher Spin Self-dual Yang-Mills

Let's take $\omega^{A(2s-2)}$ and $\Psi^{A(2s)}$ and let them take values in some (matrix) Lie algebra, then the action

$$S = \sum_n \mathrm{tr} \int \Psi^{A(2s)} H_{AA} \wedge F_{A(2s-2)}$$

where all A's are symmetrized inside F

$$F = d\omega + \omega \wedge \omega \qquad\qquad \omega = \sum_n (\omega^{A(2s)})^i{}_j \; y_A ... y_A$$

is invariant under (thanks to $H_{AA} e_{AB'} \equiv 0$)

$$\delta\omega = \nabla\xi + [\omega, \xi] \qquad\qquad \delta\omega^{A(2s-2)} = e^A{}_{C'}\, \eta^{A(2s-3),C'}$$

Feature: describes gauge, one-derivative, interactions of higher spin fields that are inaccessible via Fronsdal's approach

HS-SDGRA: Flat Space

In flat space we can simply write

$$S = \sum_{m,n} \int \boldsymbol{\Psi}^{A(n+m)}\, \boldsymbol{F}_{A(n)} \wedge \boldsymbol{F}_{A(m)}$$

where $F_{A(n)} = d\omega_{A(n)}$. To get $(A)dS_4$ we define Poisson bracket on $\mathbb{R}^2$ of $f(y)$, which is the same as $w_{1+\infty}$:

$$\{f, g\} = \epsilon^{AB}\, \partial_A f(y)\, \partial_B g(y)$$

and write $F = d\omega + \frac{1}{2}\{\omega, \omega\}$. Flat limit is smooth.

Feature: describes gravitational, two-derivative, interactions of higher spin fields that are inaccessible via Fronsdal's approach

Feature: zero-form Ψ lives in the dual space, an interesting representation of Poisson algebra

Chiral Higher Spin Gravity: Equations with/without cosmological constant

A time to unfold theories and a time to fold them (King Solomon)

Covariant Chiral Theory in flat/(A)dS

Difficult to construct actions for theories with ∞-derivative and HS symmetries, done so far for conformal HiSGRA (Basile, Grigoriev, E.S.)

Full covariant form (E.S., Sukhanov, Sharapov, Van Dongen) can be constructed following Vasiliev's commandment:

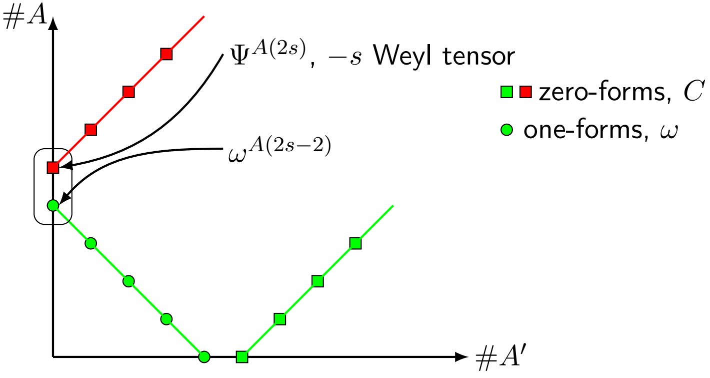
$$d\Phi = l_2(\Phi,\Phi) + l_3(\Phi,\Phi,\Phi) + ... \qquad\qquad Q^2 = 0$$

where l_n form L_∞-algebra; $\Phi = \{\omega\,, C\}$ and $\omega(y,\bar{y})$, $C(y,\bar{y})$ are generating functions that contain dynamical fields $\omega_{A(k)}$, $\Psi^{A(k)}$ and "derivatives" thereof.

$l_2(\bullet,\bullet) \approx$ higher spin algebra

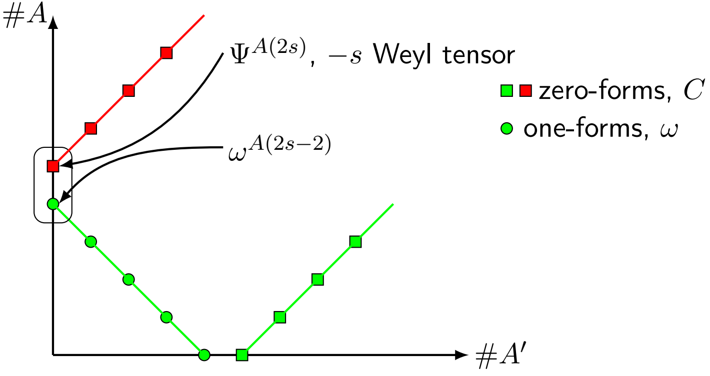
Lorentz covariant local theory with propagating massless higher spin fields. Smooth deformation to $(A)dS_4$

Covariant Chiral Theory in flat/(A)dS



One can unfold anything (Barnich, Grigoriev), why not

$$\boldsymbol{S = \int \Psi^{A(2s)} \wedge H_{AA} \wedge \nabla \omega_{A(2s-2)}}$$

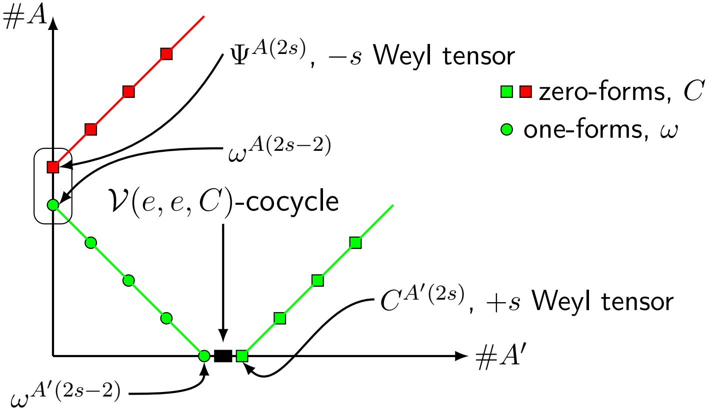


The first step leads to

$$\nabla\Psi^{A(2s)} = e_{BB'}\Psi^{A(2s)B,B'} \qquad \nabla\omega^{A(2s-2)} = e^{A}{}_{B'}\,\omega^{A(2s-3),B'}$$

In fact, the (auxiliary) fields must be the same as in (Vasiliev:86,88)

Covariant Chiral Theory in flat/(A)dS



Complete set of free equations for $\omega(x;y,\bar{y})$, $C(x;y,\bar{y})$ with λ reads

$$\nabla\omega = e^{BB'}(\boldsymbol{\lambda}\,\bar{y}_{B'}\partial_B + y_B\bar{\partial}_{B'})\,\omega + H^{B'B'}\bar{\partial}_{B'}\bar{\partial}_{B'}C(y=0,\bar{y})\,,$$

$$\nabla C = e^{BB'}(\boldsymbol{\lambda}\,y_B\bar{y}_{B'} - \partial_B\bar{\partial}_{B'})\,C\,.$$

Successful recipe

We need to fix all structure maps in

$$\begin{aligned} d\omega &= \mathcal{V}(\omega,\omega)+\mathcal{V}(\omega,\omega,C)+\mathcal{V}(\omega,\omega,C,C)+\dots\,,\\ dC &= \mathcal{U}(\omega,C)+\mathcal{U}(\omega,C,C)+\dots\,. \end{aligned}$$

Let $A_{\boldsymbol{\lambda}}(y)$ be Weyl algebra $[y_A,y_B]=\boldsymbol{\lambda}\,\epsilon_{AB}$, then

$\mathfrak{hs}$-algebra for $\boldsymbol{\lambda}=0$ must be $A_0(y)\otimes A_1(\bar{y})$ (Krasnov, E.S.), $A_0=\mathbb{C}[y]$

The finite-dim subalgebra is not Poincare, but it acts as Poincare

$$\begin{aligned} [L_{AB},L_{CD}] &= \boldsymbol{\lambda}\,\epsilon_{..}L_{..} & [\bar{L}_{A'B'},\bar{L}_{C'D'}] &= \epsilon_{..}\bar{L}_{..}\\ [L_{AA},P_{BB'}] &= \boldsymbol{\lambda}\,\epsilon_{AB}P_{AB'} & [\bar{L}_{A'A'},P_{BB'}] &= \epsilon_{..}P_{..}\\ [P_{AA'},P_{BB'}] &= \epsilon_{A'B'}L_{AB}+\boldsymbol{\lambda}\,\epsilon_{AB}\bar{L}_{A'B'} \end{aligned}$$

L_{AB} **behaves as a central charge at** $\boldsymbol{\lambda}=0$, see (Ponomarev)

Successful recipe

We need to fix all structure maps in

$$\begin{aligned} d\omega &= \mathcal{V}(\omega,\omega) + \mathcal{V}(\omega,\omega,C) + \mathcal{V}(\omega,\omega,C,C) + \dots\,, \\ dC &= \mathcal{U}(\omega,C) + \mathcal{U}(\omega,C,C) + \dots\,. \end{aligned}$$

Let $A_\lambda(y)$ be Weyl algebra $[y_A, y_B] = \lambda\epsilon_{AB}$, then

$\mathfrak{hs}$-algebra for $\boldsymbol{\lambda} = 0$ must be $A_0(y) \otimes A_1(\bar{y})$ (Krasnov, E.S.), $A_0 = \mathbb{C}[y]$

$\mathfrak{hs}$-algebra for $\boldsymbol{\lambda} \neq 0$ must be $A_{\boldsymbol{\lambda}}(y) \otimes A_1(\bar{y})$ (Sharapov, E.S.), so apart from $\boldsymbol{\lambda}$ it is the usual Dirac's $A_1 \otimes A_1$ that acts on the tensor product of singletons (Flato, Fronsdal)

As always, we can take a matrix extension $\mathfrak{hs} \to \mathfrak{hs} \otimes \mathrm{Mat}_M$. In fact, our construction is blind to the other factors, i.e. $\mathfrak{hs} = A_{\boldsymbol{\lambda}} \otimes B$, where B is any associative algebra. We always multiply with respect to B.

Successful recipe

We need to fix all structure maps in

$$
\begin{aligned}
d\omega &= \mathcal{V}(\omega,\omega) + \mathcal{V}(\omega,\omega,C) + \mathcal{V}(\omega,\omega,C,C) + \ldots\,,\\
dC &= \mathcal{U}(\omega,C) + \mathcal{U}(\omega,C,C) + \ldots\,.
\end{aligned}
$$

Effectively: $\mathfrak{hs}$-algebra is $A_{\boldsymbol{\lambda}}(y)$, which defines $\mathcal{V}(a,b)$.

The action story teaches us that the module structure $\mathcal{U}(a,C)$ is that of the dual one. For example, for $\boldsymbol{\lambda}=0$ we have $\mathfrak{hs}=\mathbb{C}[y]$ and it acts on $\mathbb{C}[y]$ by differential operators

$$
\mathcal{U}(\omega,C) = \omega(\partial_y)C(y)
$$

For $\boldsymbol{\lambda}=1$ the dual module structure coincides with the **twisted-adjoint** one, of course. **We have a smooth flat limit/deformation to $(A)dS$!**

Successful recipe

We need to fix all structure maps in

$$\begin{aligned} d\omega &= \mathcal{V}(\omega,\omega) + \mathcal{V}(\omega,\omega,C) + \mathcal{V}(\omega,\omega,C,C) + \dots\,, \\ dC &= \mathcal{U}(\omega,C) + \mathcal{U}(\omega,C,C) + \dots\,. \end{aligned}$$

The cubic vertices are certain cocycles of $\mathfrak{hs}$-algebra and can be found by hand (Sharapov, E.S., Van Dongen).

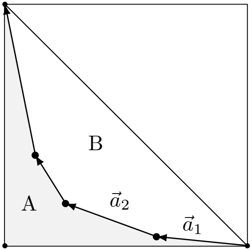
$\omega = \sum \omega_{A\dots A}\, y^A \dots y^A$ and vertices can be represented as poly-differential operators. We work with symbols thereof

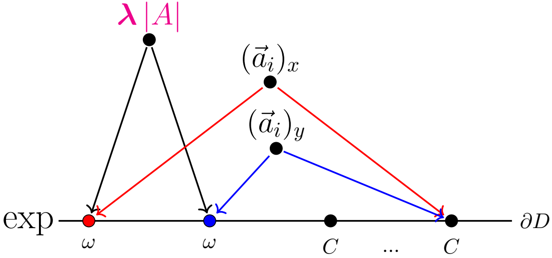
$$p_0 \equiv y \qquad\qquad p_i \equiv \partial_{y_i} \qquad\qquad p_{ij} = \epsilon_{AB}\, p_i^A p_j^B$$

e.g., the star-product's symbol is $(f \star g)(y) = e^{\boldsymbol{\lambda}\, p_{12}}\, f(y+y_1) g(y+y_2)|_{y_i=0}$

$$(f \star g)(y) = \exp[p_{01} + p_{02} + \boldsymbol{\lambda}\, p_{12}]\, f(y_1) g(y_2)\Big|_{y_i=0}$$

Explicit answer for all vertices, e.g. $\mathcal{V}(\omega,\omega,C,...,C)$:





(Sharapov, E.S., Van Dongen) the configuration space is of convex polygons B or swallowtails A, related to Grassmannian. The maps exponentiate like for Moyal-Weyl

Surprise: All vertices are local and are known!

$$\mathcal{V}(\omega,\omega,C,...,C)=(p_{12})^n\exp[*p_{01}+*p_{02}+*\boldsymbol{\lambda}\,p_{12}+\sum *p_{1i}+\sum *p_{2i}]$$

Plug and Play higher spin theory

We see a small piece of a bigger formality where $\pi^{AB} =$ const, so only boundary 'Kontsevich' graphs survive. 1st layer = Kontsevich Formality (Moyal-Weyl); 2nd layer Shoikhet-Tsygan-Kontsevich Formality

Our A_∞ is a pre-Calabi-Yau algebra (Kontsevich, ...). Lots of vertices are related to each other by **duality map** that gives local vertices

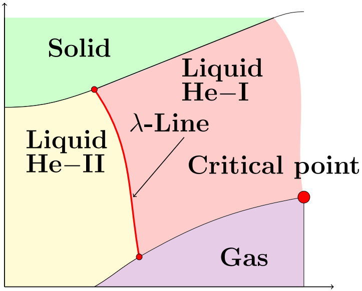
$$\langle \mathcal{V}(\omega,\omega,C,\ldots,C)|C\rangle = \langle \omega|\mathcal{U}(\omega,C,\ldots,C)\rangle$$

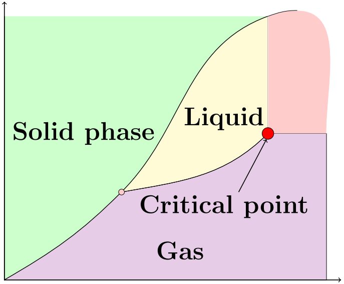
Equations of motion are of a $2d/4d$ Poisson Sigma Model

$$dC^i = \pi^{ij}(C)\,\omega_j\,, \qquad\qquad d\omega_k = \tfrac{1}{2}\partial_k\pi^{ij}(C)\,\omega_i\,\omega_j\,.$$

Absolute reference frame (HS Eather): maximal locality vs. L_∞/Q-manifold?

Chern-Simons Matter Theories and bosonization duality





Chern-Simons Matter theories and dualities

In AdS_4/CFT_3 one can do much better — there exists a large class of models, Chern-Simons Matter theories (extends to ABJ(M))

$$\frac{k}{4\pi}S_{CS}(A) + \text{Matter}\begin{cases}(D\phi^i)^2 & \textbf{free boson}\\ (D\phi^i)^2 + g(\phi^i\phi^i)^2 & \textbf{Wilson-Fisher (Ising)}\\ \bar{\psi}\not{D}\psi & \textbf{free fermion}\\ \bar{\psi}\not{D}\psi + g(\bar{\psi}\psi)^2 & \textbf{Gross-Neveu}\end{cases}$$

- describe physics (Ising, quantum Hall, ...), break parity
- two parameters $\lambda = N/k$, $1/N$ (λ continuous for N large)
- exhibit remarkable dualities, e.g. **$3d$ bosonization duality** (Aharony, Alday, Bissi, Giombi, Karch, Maldacena, Minwalla, Prakash, Seiberg, Tong, Witten, Yacobi, Yin, Zhiboedov, ...)

Chern-Simons Matter theories and dualities

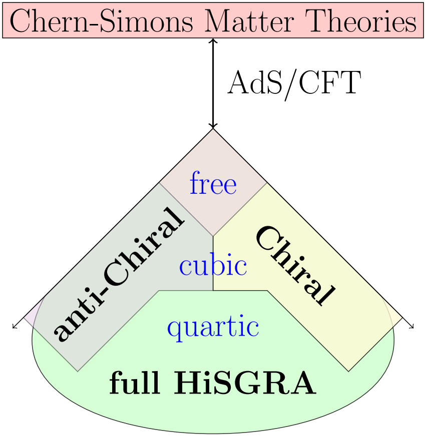
$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\textbf{free boson} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"'] & \textbf{Gross-Neveu} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"] \\
\textbf{Wilson-Fisher} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] & \textbf{free fermion} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"]
\end{tikzcd}
$$

The simplest gauge-invariant operators are higher spin currents

$$J_s = \phi D ... D \phi \qquad\qquad J_s = \bar{\psi} \gamma D ... D \psi$$

which are AdS/CFT dual to higher spin fields

Chiral HiSGRA and Chern-Simons Matter



(anti)-Chiral Theories are rigid;
$\exists$ **closed subsector of CS-matter**

gluing introduces one parameter via EM-duality $\Phi_{\pm s} \to e^{\pm i\theta}\Phi_{\pm s}$

gives all 3-point correlators consistent with (Maldacena, Zhiboedov)

can be pushed to 4pt (E.S., Yin)

Bosonization is manifest! Concrete predictions from HiSGRA.

(anti)-Chiral Theories provide a complete base for 3-pt amplitudes

$$\boldsymbol{V_3 = V_{chiral} \oplus \bar{V}_{chiral} \quad \leftrightarrow \quad \langle JJJ \rangle}$$

Chiral Theory vs. Tensionless Strings

Chiral HSGRA vs. Tensionless Strings

Strings on $AdS_4 \times \mathbb{CP}^3$ are dual to ABJ theory = Chern-Simons (k) matter theories with bi-fundamental matter, $N \times M$, (Chang, Minwalla, Sharma, Yin)

There is a vector-like limit $N \gg M$, where it is dual to $\mathcal{N} = 6\ U(M)$-gauged HiSGRA, which is, of course, non-existing due to the non-locality

Inside this non-local/non-existing HiSGRA there is $\mathcal{N} = 6\ U(M)$-gauged Chiral HiSGRA, which is local

Is it possible to directly identify the Chiral subsector of tensionless strings on $AdS_4 \times \mathbb{CP}^3$?

Higher spin symmetry
and bosonization duality

Unbroken Higher spin symmetry

In free theories we have ∞-many conserved $J_s = \phi\partial...\partial\phi$ tensors.

Free CFT $=$ Associative (higher spin) algebra

Conserved tensor $\rightarrow$ current $\rightarrow$ symmetry $\rightarrow$ invariants$=$correlators.

$$\partial \cdot J_s = 0 \quad \Longrightarrow \quad Q_s = \int J_s \quad \Longrightarrow \quad [Q,Q] = Q \quad \& \quad [Q,J] = J$$

HS-algebra (free boson) $=$ HS-algebra (free fermion) in $3d$.

Correlators are given by invariants (Sundell, Colombo; Didenko, E.S.; ...)

$$\langle J...J\rangle = \mathrm{Tr}(\Psi \star ... \star \Psi) \qquad\qquad \Psi \leftrightarrow J$$

where Ψ are coherent states representing J in the higher spin algebra

$$\langle JJJJ\rangle_{F.B.} \sim \cos(Q^2_{13} + Q^3_{24} + Q^4_{31} + Q^1_{43}) \cos(P_{12}) \cos(P_{23}) \cos(P_{34}) \cos(P_{41}) + ...$$

Slightly-broken Higher spin symmetry is new Virasoro?

In large-N Chern-Simons vector models (e.g. Ising) higher spin symmetry does not disappear completely (Maldacena, Zhiboedov):

$$\partial \cdot J = \frac{1}{N}[JJ] \qquad\qquad [Q, J] = J + \frac{1}{N}[JJ]$$

What is the right math? We should deform the algebra together with its action on the module, so that the currents can 'backreact':

$$\delta_\xi J = l_2(\xi, J) + l_3(\xi, J, J) + \ldots\,, \qquad\qquad [\delta_{\xi_1}, \delta_{\xi_2}] = \delta_\xi\,,$$

where $\xi = l_2(\xi_1, \xi_2) + l_3(\xi_1, \xi_2, J) + \ldots$. This leads to L_∞-algebra.

Correlators = invariants of L_∞-algebra and are unique (Gerasimenko, Sharapov, E.S.), **which proves $3d$ bosonization duality at least in the large-N**. Without having to compute anything one prediction is

$$\langle J \ldots J \rangle = \sum \langle \text{fixed} \rangle_i \times \text{params}$$

Future Chiral Prospects and Summary

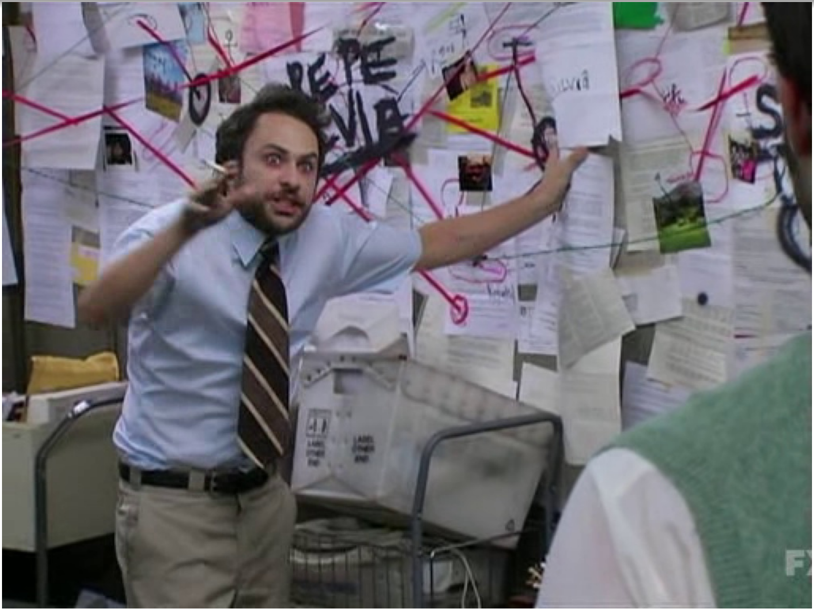
Future Prospects and Summary

- Choose 2 out of 3: higher spins, locality, unitarity?
- Everything in Chiral Theory is unitary (belongs to a unitary theory)
- Finite/one-loop exact? Twistor action (Tran)? Big theory?
- Chiral Theory is effectively $2d$, PSM, integrability (Ponomarev)
- Relation to tensionless strings on $AdS_4 \times \mathbb{CP}^3$?
- Correlators $\in$ CS-matter. Its existence suggests $3d$ dualities. Finite N? Exact AdS/CFT?
- Slightly-broken HS $=$ new physical symmetry, L_∞. Uniqueness of invariants suggests $3d$ bosonization
- Exact solutions: black-hole singularity resolution, instantons, cosmological solutions, ...
- Massive HS interactions (Ochirov, E.S.), Black-Hole scattering
- Chiral Theory detected via celestial OPE (Ren, Spradlin, Srikant, Volovich)

Thank you for your attention!

May the higher spin force be with you

1 minute after locality of non-local HiSGRA is understood



Massive Higher Spins

(no no-go's, no challenge?)

Massive higher spins?

... but string's spectrum is full of massive higher spins ...

Massive higher spins are notoriously more complicated: second class constraints, Boulware-Deser ghosts, actions are not easy (Singh, Hagen; Zinoviev)

$$(\Box - m^2)\Phi_{\mu_1\dots\mu_s} = 0 \qquad\qquad \partial^\nu \Phi_{\nu\mu_2\dots\mu_s} = 0$$

Low spins: $s = 1$ spontaneously broken Yang-Mills; $\boldsymbol{s = 3/2}$; $s = 2$ massive (bi)-gravity (dRGT; Hassan, Rosen)

Simple idea in $4d$ (Ochirov, E.S.): instead of $\Phi_{A(s),A'(s)}$, i.e. (s,s) of $sl(2,\mathbb{C})$ we suggest chiral description $\Phi_{A_1\dots A_{2s}}$, i.e. $(2s,0)$. Parity is not easy ...

Easy to introduce EM, YM and gravitational interactions, all-helicity-plus amplitudes are reproduced; relation to black-hole scattering (Arkani-Hamed, Huang^2; Guevara, Ochirov, Vines). Of course, these are effective field theories ... but everything small and rotating is a higher spin particle from a distance

Self-dual Gravity

Let us start with the frame-like gravity ($H^{AB} \equiv e^{A}{}_{C'} \wedge e^{BC'}$)

$$S = \int H_{AA} \wedge R^{AA} + \tfrac{1}{2}\Lambda\, H_{AA}H^{AA}\,, \qquad R = d\omega^{AA} + \omega^{A}{}_{B} \wedge \omega^{BA}$$

If we want to make H^{AA} an independent field, we have to remember $H^{AA} \wedge H^{AA} = 0$, which can be imposed via

$$S[\omega, H, \Psi] = \int H_{AA} \wedge R^{AA} + \tfrac{1}{2}\Lambda\, H_{AA}H^{AA} + \tfrac{1}{2}\Psi^{AAAA}H_{AA}H_{AA}$$

Now we solve for H via $R + (\Lambda + \Psi)H = 0$ and expand in Ψ to get

$$S[\omega, \Psi] = \int R^{AA} \wedge R_{AA} + \boldsymbol{\Psi^{AAAA}\, R_{AA} \wedge R_{AA}} + ...$$

The first term is topological, the second is SDGRA (Krasnov). Dropping ω^2 we get SDGRA in flat $\int \boldsymbol{\Psi^{AAAA}\, d\omega_{AA} \wedge d\omega_{AA}}$ (E.S., Krasnov)